Planar Segmentation Using Range Images From Terrestrial Laser Scanning

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***Abstract*—Various methods are available for the planar segmentation of point clouds from terrestrial laser scanning. In this letter, a new method is proposed to extract planar features from the range image of a point cloud scanned from one standpoint. In this method, a plane is parameterized by its normal vector and the distance from the origin. The algebraic derivation of the parameters is presented in this letter. The parameters are calculated based on the gradient value of a pixel in the range image. The multiple-band synthetic image of planar parameters is segmented using the Iso cluster unsupervised classification method. Experimental plane segmentation results using range images of two point clouds are illustrated. In comparison with existing methods, the proposed method gives an exact estimation of the planar parameters and can handle planes of any orientation.**

***Index Terms*—Planar segmentation, range image, terrestrial laser scanning.**

I. Introduction

Terrestrial laser scanning has become an important tool to acquire detailed 3-D geometric structures of various types of objects, such as archaeological sites [1], buildings [2], trees [3], etc. Point clouds obtained from terrestrial laser scanning are an unorganized and unstructured data set of huge volumes, which in its raw form can be a great challenge to deal with. For many applications, the segmentation of point clouds is an essential step before final products can be derived [4], [5].

Planar features are ubiquitous in man-made structures such as buildings [2] and industrial installations [6]. Planar features in point clouds can be used for the 3-D reconstruction of buildings [7], indoor navigation [8], [9], identification of planar targets [10], registration of point clouds [11], etc. Planar features can be extracted from point clouds using segmentation algorithms. There are a number of planar segmentation algorithms, and they can generally be classified as region-growing methods and clustering methods [4], [6], [12].

Any planar segmentation algorithm requires a certain type of parameter space to be defined for planes. A plane is uniquely characterized by its normal vector and its distance from the origin. Various methods can be employed to extract planar parameters from point clouds. Vosselman *et al.* [10] used the 3-D Hough transform to extract planes. The parameter space of a plane is discretized, and the bin containing the highest number of points is used to estimate the planar parameters. He *et al.* [13] proposed a line-based spectral clustering method for planar structure extraction. Straight line segments are derived from point clouds, and planar features are detected by the mean-shift clustering algorithm. A widely used method for plane extraction from point clouds is the principal component analysis (PCA). In this method, the covariance matrix of a group of adjacent points is calculated, and the eigenvector corresponding to the smallest eigenvalue of the covariance matrix is the normal vector of the fitted plane [11], [15]–[17]. Extracting planes directly from point clouds presents some difficulties. First, a point cloud can contain tens of millions of points [18]. The massive volume of a point cloud presents a challenge for its manipulation because of the processing limitations on personal computers [4]. Second, the search for neighboring points is required for the widely used PCA method. The high computation cost incurred in the neighborhood search is a big concern for any large-scale LiDAR application [10]. Another method for planar parameter extraction is based on range images. The point cloud scanned at each station is a 2-D systematic sampling of objects in the 3-D space viewed from one standpoint. In this way, a range image is an equivalent representation of the point cloud, irrespective of the intensity information. Compared with the PCA method, working with range images avoids the neighborhood search in the 3-D space. In addition, a 2-D range image is much easier to manipulate on a PC than 3-D point clouds. Many standard techniques are also available for feature extraction from images. Features extracted from different range images can be used for many purposes. Eysn *et al.* [19] extracted tree skeletons from different range images and merged extracted skeletons to form a complete tree skeleton, achieving the same effect of skeleton extraction from the full point cloud. Key points and planes extracted from multiple range images can be used for the registration of multiple scans [15], [18], [20], [21]. Planes extracted from single scans can be further processed in a similar way.

Gorte [22] and his collaborators [8] proposed a gradient-based planar segmentation method using range images. In their method, a plane is parameterized by its distance from the origin and the horizontal and vertical angles of its normal vector. The parameters of a plane are derived based on geometric relationships. In this letter, we show that the angles computed using Gorte's method are not exact values, but only approximate values. Their method also has difficulty in dealing with horizontal planes. In this letter, we propose a planar segmentation method from range images, which gives an exact estimation of the planar parameters and can deal with planes of any arbitrary orientation without any difficulty.

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The remainder of this letter is organized as follows. Section II gives an introduction on range image generation and a review of planar segmentation based on range images. In Section III, we present our method for planar segmentation from range images. Section IV shows some experimental results using the proposed method. Section V concludes this letter.

II. Literature Review

A. Range Image Generation

The dimensions of a range image are specified by four parameters: number of rows, number of columns, horizontal angular resolution, and vertical angular resolution. Suppose that a point has coordinates (x, y, z) in the Cartesian coordinate system and its spherical coordinates are (R, α, β), where R is the distance between the point and the origin, α is the horizontal angle, and β is the vertical angle. The Cartesian coordinates can be derived from the spherical coordinates using

$$x = R \cos\beta \cos\alpha \tag{1}$$

$$y = R \cos\beta \sin a \tag{2}$$

$$z = R \sin\beta \tag{3}$$

and the spherical coordinates can be derived from the Cartesian coordinates using

$$R = sqrt(x^2 + y^2 + z^2) \tag{4}$$

$$\beta = \arcsin\left(\frac{z}{R}\right) \tag{5}$$

$$\alpha = \text{atan2}(y, x). \tag{6}$$

The function atan2 in (6) is the arctangent function found in many computer languages, and it returns the appropriate quadrant of the computed angle [23].

A pixel in a range image is specified by a coordinate pair (r, c), where r is the row index and c is the column index. They are calculated as

$$c = \frac{\alpha}{\Delta\alpha} \tag{7}$$

where $\Delta\alpha$ is the horizontal angular resolution and

$$r = \frac{\beta}{\Delta\beta} \tag{8}$$

where $\Delta\beta$ is the vertical angular resolution.

Due to uncertainties in the range and angular resolutions in terrestrial laser scanning, each scanned point has limited positional accuracy. As a result, two points may fall within one pixel in the range image, and the value of the pixel is aggregated from all points within it [19].

B. Planar Parameter Estimation

To extract planes from a range image, the parameters of the plane containing a given pixel need to be estimated from the image. A plane can be described using

$$n_x x + n_y y + n_z z + \rho = 0 \tag{9}$$

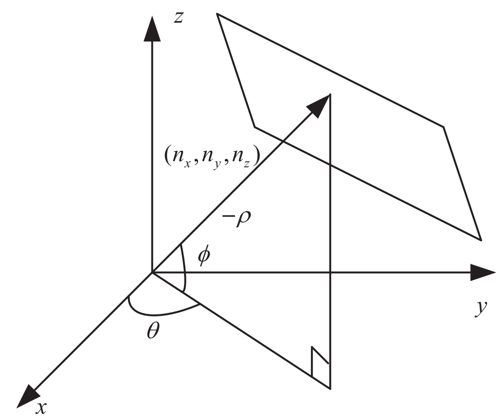

Fig. 1. Planar parameterization using the three components of the normal vector versus using the horizontal and vertical angles of the normal vector.

where (n_x, n_y, n_z) is the unit normal vector of the plane, (x, y, z) is a point on the plane, and ρ is the signed distance between the origin and the plane. The normal vector of a plane is determined by (θ, ϕ), where θ is the horizontal angle and ϕ is the vertical angle.

Gorte and his collaborators [8], [22] proposed a method to obtain the values of (θ, ϕ). In their method, the value of θ is obtained using

$$\theta = \alpha - \text{atan}\left(\frac{1}{R}\frac{\partial R}{\partial \alpha}\right). \tag{10}$$

In the following, we prove that the angle obtained by (10) is not the true value of θ, but an approximate value to the true value of θ. The unit normal vector (n_x, n_y, n_z) can be expressed using the horizontal and vertical angles and becomes $(\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi)$ (see Fig. 1).

From (1)–(3) and (9), the plane equation becomes

$$R\cos\beta\cos\alpha\cos\theta\cos\phi + R\cos\beta\sin a\sin\theta\cos\phi + R\sin\beta\sin\phi + \rho = 0. \tag{11}$$

Using trigonometric identities and rearranging the terms of (11), we obtain

$$-\frac{\rho}{R} = \cos\beta\cos\phi\cos(\alpha - \theta) + \sin\beta\sin\phi. \tag{12}$$

Taking partial derivatives with respect to α on both sides, we obtain

$$\frac{\rho}{R^2}\frac{\partial R}{\partial \alpha} = -\sin(\alpha - \theta)\cos\beta\cos\phi. \tag{13}$$

From (12) and (13), we obtain

$$\frac{1}{R}\frac{\partial R}{\partial \alpha} = \tan(\alpha - \theta)\left(1 - \frac{\sin\beta\sin\phi}{\cos(\alpha-\theta)\cos\beta\cos\phi + \sin\beta\sin\phi}\right). \tag{14}$$

From (10), we have

$$\frac{1}{R}\frac{\partial R}{\partial \alpha} = \tan(\alpha - \theta). \tag{15}$$

Comparing (14) and (15), we conclude that the angle obtained from (14) is not equal to the true value of θ.

If we want to derive the value of θ using only derivatives with respect to α, we can take derivatives on both sides of (13) and obtain

$$\frac{\rho}{R^2}\frac{\partial^2 R}{\partial\alpha^2} - \frac{2\rho}{R^3}\left(\frac{\partial R}{\partial\alpha}\right)^2 = -\cos(\alpha-\theta)\cos\beta\cos\phi. \quad (16)$$

From (13) and (16), we have

$$\tan(\alpha-\theta) = \frac{\frac{\partial R}{\partial\alpha}}{\frac{\partial^2 R}{\partial\alpha^2} - \frac{2}{R}\left(\frac{\partial R}{\partial\alpha}\right)^2}. \quad (17)$$

Using (14) or (17) to compute the value of θ suffers from several problems. The first problem is that the values of θ obtained using the $a\tan$ trigonometric function may differ by a value of π and there is no easy way to determine which one to use. The second problem is that the value of $\tan(\alpha-\theta)$ can be an infinity and thus cannot be computed. For horizontal planes, the value of θ is not well defined, which can complicate the following segmentation procedure [22]. Finally, the second-order derivatives in (17) computed from a range image are more sensitive to noises in range images than first-order derivatives.

In the following section, we propose a new way to calculate the parameters of a plane using range images. The new method does not suffer from the aforementioned problems.

III. Planar Segmentation Methodology

A. Planar Parameterization Using the Normal Vector

Instead of using (θ,ϕ) to parameterize the normal vector, we directly compute the three components of the normal vector (n_x, n_y, n_z) from the range image. Suppose that g_x is the horizontal component of the gradient and g_y is the vertical component of the gradient. We have from (1)–(3) and (9)

$$-\frac{\rho}{R} = n_x\cos\beta\cos\alpha + n_y\cos\beta\sin a + n_z\sin\beta. \quad (18)$$

Taking partial derivatives with respect to α and β on both sides, we obtain

$$\frac{\rho}{R^2}g_x = -n_x\cos\beta\sin\alpha + n_y\cos\beta\cos a \quad (19)$$

$$\frac{\rho}{R^2}g_y = -n_x\sin\beta\cos\alpha - n_y\sin\beta\sin a + n_z\cos\beta. \quad (20)$$

From (18) and (20), we have

$$n_z = \frac{\rho}{R^2}(g_y\cos\beta - R\sin\beta) \quad (21)$$

$$n_x\cos\alpha + n_y\sin a = -\frac{\rho}{R^2}(g_y\sin\beta + R\cos\beta). \quad (22)$$

From (19) and (22), we have

$$n_x = -\frac{\rho}{R^2}\left(\cos\alpha(g_y\sin\beta + R\cos\beta) + \frac{g_x\sin\alpha}{\cos\beta}\right) \quad (23)$$

$$n_y = -\frac{\rho}{R^2}\left(\sin\alpha(g_y\sin\beta + R\cos\beta) - \frac{g_x\cos\alpha}{\cos\beta}\right). \quad (24)$$

From (21), (23), and (24), we have

$$n_x^2 + n_y^2 + n_z^2 = \frac{\rho^2}{R^4}\left(g_y^2 + R^2 + \frac{g_x^2}{\cos^2\beta}\right). \quad (25)$$

Because the normal vector is a unit vector, we have

$$\rho = \frac{R^2}{sqrt\left(R^2 + g_y^2 + \frac{g_x^2}{\cos^2\beta}\right)} \quad (26)$$

where only the positive value of ρ is used for further processing because the negative value of ρ, along with the corresponding normal vector, defines the same plane as the positive value of ρ.

Equations (21), (23), (24), and (26) are used to derive the parameters of the plane at each point corresponding to the pixel in a range image. The parameters consist of the normal vector and the distance from the origin.

B. Gradient Computation of Range Images

In order to extract planar parameters using the proposed method, the gradient at each pixel of the range image needs to be calculated. Many operators, such as the Sobel kernel, can be used to calculate the gradients. To alleviate the problem of positional uncertainties and noises in the point cloud, we use filters with a longer support than the Sobel kernel. The choice of the support of filters should take such factors as angular resolution and noises into consideration. Filters of shorter support are more sensitive to noises, and filters of longer support tend to smooth the edges between adjacent clusters.

The horizontal filter H used in this letter is specified as

$$H = [-1, -2, -3, -4, 0, 4, 3, 2, 1] \quad (27)$$

and the vertical filter V is specified as

$$V = [1, 2, 3, 4, 0, -4, -3, -2, -1]^T. \quad (28)$$

The filters give more weights to pixels closer to the center pixel. The filters have a support of medium length. The horizontal component of the gradient at a given pixel equals the convoluted value divided by the term of $40\Delta\alpha$, and the vertical component equals the convoluted value divided by the term of $40\Delta\beta$. The denominators serve to derive the gradients from the convoluted values.

C. Range Image Segmentation

By calculating the planar parameters at each pixel, we obtain a synthetic image of planar parameters that can be used for planar feature segmentation. The synthetic image is a four-band image, and each parameter corresponds to one band. There are many image segmentation methods that can be used, such as the region-growing method [6], K-means clustering method, fuzzy segmentation [4], Iso cluster unsupervised classification [24], etc. The segmentation results depend on many factors, including the segmentation method, maximum number of clusters, maximum number of iterations, and minimum number of pixels in a cluster. The Iso cluster unsupervised classification method is readily available in ArcGIS, and it is used to conduct the unsupervised segmentation of the planar parameter image in this letter for the sake of convenience.

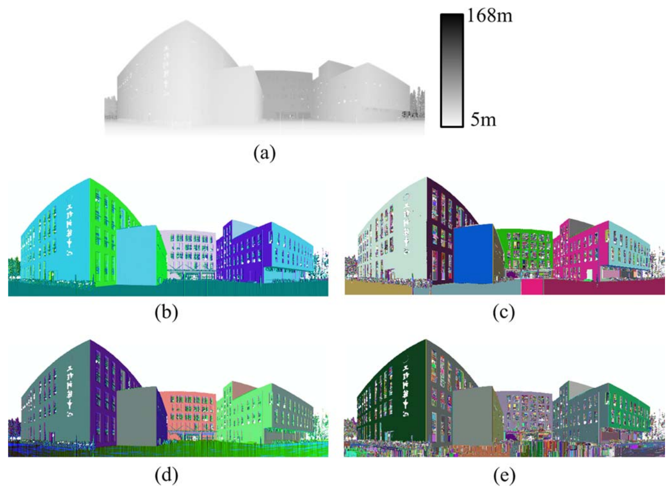

Fig. 2. Planar segmentation of a building. (a) Range image. (b) Synthetic image of planar parameters (ρ, n_x, n_y). (c) Segmented image using the synthetic image in (b). (d) Synthetic image of planar parameters (ρ, θ, ϕ) using the method in [8]. (e) Segmented image using the synthetic image in (d).

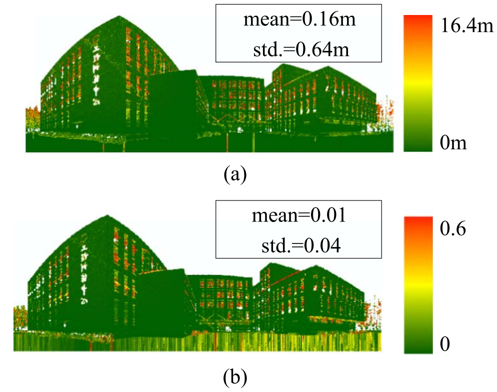

Fig. 3. Difference images of planar parameters computed using our proposed method and the method in [8]. (a) Difference image of planar parameter ρ. (b) Difference image of the normal vectors. The lengths of the difference normal vectors are shown.

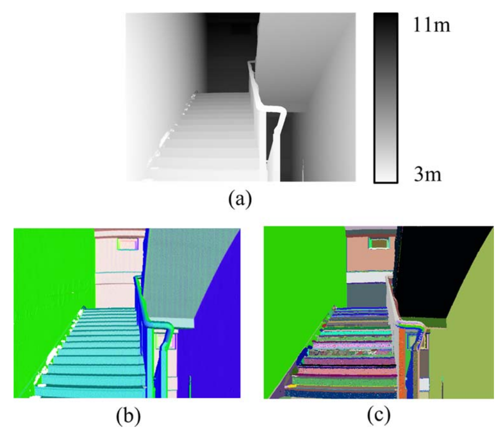

Fig. 4. Planar segmentation of a staircase. (a) Range image. (b) Synthetic image of planar parameters (ρ, n_x, n_y). (c) Segmented image using the synthetic image in (b).

IV. Experimental Results

The proposed planar segmentation method is employed to two point clouds to extract planar features. The point clouds are obtained using a Leica ScanStation C10 laser scanner. The two point clouds are scanned with the medium angular resolution, which has the spacing of 0.1 m at a distance of 100 m. Both the horizontal and vertical angular resolutions are taken as 0.057° in the experiments. The maximum number of clusters is set to be 15, and the minimum number of cells in a cluster is set to be 50. All bands in the synthetic images are normalized to the range of 0–1. Due to noises contained in the point cloud, the range values are averaged if more than one point falls within a single grid cell. For those cells that do not correspond to any point, their range values are marked as NODATA, and they will be skipped over in subsequent processing. Because disjoint clusters may be assigned the same label in a segmented image by the unsupervised classification method, a recoding postprocessing operation is applied to the segmented image so that each disjoint cluster has a unique label.

The first point cloud is a building of the library complex on the campus of the university with which the authors are affiliated and contains approximately 2 400 000 points. The horizontal angle ranges from 150° to 310°, and the vertical angle ranges from $-10°$ to 60°. The range image and segmentation result are shown in Fig. 2. The range image is shaded in the way that short ranges correspond to light shades and long ranges correspond to dark shades. Clusters are differentiated using different colors. The synthetic image of the three planar parameters (ρ, θ, ϕ) using the method in [8] and the corresponding segmented image are also shown in Fig. 2. As can be seen clearly, compared with our proposed method, the horizontal planes extracted using the method in [8] are oversegmented because the horizontal angle is not well defined. The oversegmentation of horizontal planes is also demonstrated in [22].

Fig. 3 shows the difference images of planar parameter ρ and the normal vectors computed from the first point cloud using our proposed method and the method in [8]. It can be seen from Fig. 3 that the differences of ρ are very small for points located on planar surfaces such as the ground and the facades of the building. Large differences of ρ occur at points not located on planar surfaces. The differences between the normal vectors are very small for points located on the facades of the building, whereas the difference between the normal vectors is relatively large for points on the ground. This is due to the fact that the angles on the horizontal planes are not well defined.

The second point cloud is a staircase and contains approximately 7 250 000 points. The horizontal angle ranges from 195° to 255°, and the vertical angle ranges from $-10°$ to 35°. The point cloud contains planes of various orientations, including walls and stair steps. The range image and segmentation result are shown in Fig. 4.

In both experiments, the horizontal planes are satisfactorily segmented from other features, whereas the method proposed by Gorte and his collaborators [8], [22] has difficulty in extracting horizontal planes. The segmentation results, however, depend on many factors, including the gradient filter, the number of clusters predefined, the minimum number of pixels each that cluster contains, and the segmentation method. Postsegmentation image processing techniques, such as majority filtering and group clumping, may be applied to make the segmentation results more suitable for further processing.

V. Conclusion

Various methods are available for the planar segmentation of point clouds from terrestrial laser scanning. A range image is an equivalent representation of a point cloud scanned from one standpoint. In this letter, a new method is proposed to extract planar features from the range image of a point cloud scanned from one standpoint. In this method, a plane is parameterized by its normal vector and the distance from the origin. Unlike using horizontal and vertical angles to parameterize the normal vector of a plane, using the components of a normal vector enables the method to handle planes of any orientation. The algebraic derivation of the parameters is presented. The parameters are calculated based on the gradient value of a pixel in the range image. The Iso cluster unsupervised classification method is used to segment the synthetic images of planar parameters. Experimental plane segmentation results using range images of two point clouds are illustrated. In comparison with existing methods for planar parameterization, the proposed method gives an exact estimation for the planar parameters and can handle planes of any orientation.

The proposed planar segmentation method is based on range images, and the planar parameters are calculated using gradients, which can be calculated very efficiently using the standard image processing method. As with any segmentation method, the proposed planar segmentation method is affected by many factors, such as the filtering strategy, segmentation method, maximum number of clusters, maximum number of iterations, and minimum number of pixels in a cluster. The method is applicable to point clouds scanned from one single standpoint. Planes segmented from multiple standpoints can be used for such applications as plane-based registration and 3-D reconstruction of buildings.

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References

[1] J. L. Lerma, S. Navarro, A. E. Seguí, and M. Cabrelles, "Range-based versus automated markerless image-based techniques for rock art documentation," *Photogramm. Rec.*, vol. 29, no. 145, pp. 30–48, Mar. 2014.

[2] P. Tang, D. Huber, B. Akinci, R. Lipman, and A. Lytle, "Automatic reconstruction of as-built building information models from laser-scanned point clouds: A review of related techniques," *Autom. Construction*, vol. 19, no. 7, pp. 829–843, Nov. 2010.

[3] P. Raumonen *et al.*, "Fast automatic precision tree models from terrestrial laser scanner data," *Remote Sens.*, vol. 5, no. 2, pp. 491–520, Jan. 2013.

[4] J. M. Biosca and J. L. Lerma, "Unsupervised robust planar segmentation of terrestrial laser scanner point clouds based on fuzzy clustering methods," *ISPRS J. Photogramm. Remote Sens.*, vol. 63, no. 1, pp. 84–98, Jan. 2008.

[5] B. Yang and Z. Dong, "A shape-based segmentation method for mobile laser scanning point clouds," *ISPRS J. Photogramm. Remote Sens.*, vol. 81, no. 7, pp. 19–30, Jul. 2013.

[6] T. Rabbani, S. Dijkman, F. van den Heuvel, and G. Vosselman, "An integrated approach for modeling and global registration of point clouds," *ISPRS J. Photogramm. Remote Sens.*, vol. 61, no. 6, pp. 355–370, Feb. 2007.

[7] S. Pu and G. Vosselman, "Knowledge based reconstruction of building models from terrestrial laser scanning data," *ISPRS J. Photogramm. Remote Sens.*, vol. 64, no. 6, pp. 575–584, Nov. 2009.

[8] B. Gorte, K. Khoshelhma, and E. Verbree, "Indoor navigation by using segmentation of range images obtained by laser scanners," *Int. Archives Photogramm., Remote Sens. Spatial Inf. Sci.*, vol. 37, pp. 971–976, 2008.

[9] K. Khoshelham, "Automated localization of a laser scanner in indoor environments using planar objects," in *Proc. Int. Conf. IPIN*, 2010, pp. 1–7.

[10] G. Vosselman, B. G. H. Gorte, G. Sithole, and T. Rabbani, "Recognising structure in laser scanner point clouds," *Int. Archives Photogramm., Remote Sens. Spatial Inf. Sci.*, vol. 36, pp. 33–38, 2004.

[11] C. Dold and C. Brenner, "Registration of terrestrial laser scanning data using planar patches and image data," *Int. Archives Photogramm., Remote Sens. Spatial Inf. Sci.*, vol. 36, pp. 78–83, 2006.

[12] J. Wang and J. Shan, "Segmentation of LiDAR point clouds for building extraction," in *Proc. ASPRS Annu. Conf.*, Baltimore, MA, USA, 2009, pp. 1–13.

[13] Y. He, C. Zhang, and C. S. Fraser,"A line-based spectral clustering method for efficient planar structure extraction from lidar data," *ISPRS Ann. Photogramm., Remote Sens. Spatial Inf. Sci.*, Antalya, Turkey, Nov. 11–13, 2013, vol. II-5/W2, pp. 103–108.

[14] Y.-B. Liang, Q.-M. Zhan, E.-Z. Che, M.-W. Chen, and D.-L. Zhang, "Automatic registration of terrestrial laser scanning data using precisely located artificial planar targets," *IEEE Geosci. Remote Sens. Lett.*, vol. 11, no. 1, pp. 69–73, Jan. 2014.

[15] C. Brenner, C. Dold, and N. Ripperda, "Coarse orientation of terrestrial laser scans in urban environments," *ISPRS J. Photogramm. Remote Sens.*, vol. 63, no. 1, pp. 4–18, Jan. 2008.

[16] J. K. Seo, G. C. Sharp, and L. Sang Wook, "Range data registration using photometric features," in *Proc. IEEE Comput. Soc. Conf. CVPR*, 2005, vol. 2, pp. 1140–1145.

[17] T. Huang, D. Zhang, G. Li, and M. Jiang, "Registration method for terrestrial LiDAR point clouds using geometric features," *Opt. Eng.*, vol. 51, no. 2, Feb. 2012, Art. ID 021114.

[18] S. Barnea and S. Filin, "Keypoint based autonomous registration of terrestrial laser point-clouds," *SPRS J. Photogramm. Remote Sens.*, vol. 63, no. 1, pp. 19–35, Jan. 2008.

[19] L. Eysn *et al.*, "A practical approach for extracting tree models in forest environments based on equirectangular projections of terrestrial laser scans," *Remote Sens.*, vol. 5, no. 11, pp. 5424–5448, Oct. 2013.

[20] Z. Kang, J. Li, L. Zhang, Q. Zhao, and S. Zlatanova, "Automatic registration of terrestrial laser scanning point clouds using panoramic reflectance images," *Sensors*, vol. 9, no. 4, pp. 2621–2646, Apr. 2009.

[21] G. Zhou, B. Wang, and J. Zhou, "Automatic registration of tree point clouds from terrestrial lidar scanning for reconstructing the ground scene of vegetated surfaces," *IEEE Geosci. Remote Sens. Lett.*, vol. 11, no. 9, pp. 1654–1658, Sep. 2014.

[22] B. Gorte, "Planar feature extraction in terrestrial laser scans using gradient based range image segmentation," *Int. Archives Photogramm., Remote Sens. Spatial Inf. Sci.*, vol. 36, pp. 173–177, 2007.

[23] A. Ukil, V. H. Shah, and B. Deck, "Fast computation of arctangent functions for embedded applications: A comparative analysis," in *Proc. IEEE ISIE*, 2011, pp. 1206–1211.

[24] " Iso Cluster Unsupervised Classification," Environ. Syst. Res. Inst. (ESRI), Redlands, CA, USA, Oct. 31, 2015. [Online]. Available http://resources.arcgis.com/en/help/main/10.2/#/Iso_Cluster_Unsupervised_Classification/009z000000pn000000/